National Center for Theoretical Sciences
Physics Division

NCTS started its operation on August 1, 1997, sponsored by grants from National Science Council, co-hosted by NTHU and NCTU.

- 1st phase, 1997-2003, two 3 year projects.
- 2nd phase, 2004-2008, a 5 year project.
- 3rd phase, 2009-2014, a 6 year project.

2015 will start the 4th phase.

You are all invited to visit NCTS for collaborations.

A Summary of Research Activities

Item	2009	2010	2011	2012	2013/7
SCI paper[1]	60	93	87	97	36
Acknowledged paper[2]	82	136	179	128	78
Impact Factor Rank Top15% SCI paper[3]	39	60	51	67	21
J Chem Phys + PRB + PRD[4]	21	27	25	23	12
Conferences and workshops	27	24	30	44	24
Summer/winter school	14	15	13	16	9
Students and Postdocs attending oversea conference /school	55	64	70	51	27
Seminar	436	421	456	425	290
International Visitors (times of visits)	177	215	205	215	160

[1]SCI paper: Published SCI paper with NCTS as an affiliation.

[2]Published SCI papers acknowledging NCTS support. These include papers published by visitors to the center and also people working with FGs.

[3]Papers published in top 15% impact factor journals listed by ISI Web of Knowledge for each relevant sub-field.

[4]This year, these journals' Impact Factor is outside top 15% in relevant ISI Web of Knowledge list. But these journals are in fact representing the leading international journals in respective fields. Papers published in these journals have significant impacts.

International Collaborations

Institutions having MOU

- Center of Excellence for Particle Physics at the Terascale (CoEPP), Australia
- College of Mathematics and Physcis, Chongqing Posts and Telecommunications University, China
- College of Physics, Peking University, China
- High Energy Accelerator Laboratory (KEK), Japan
- Yukawa Institute of Theoretical Physics (YITP), Japan
- Asia-Pacific Center for Theoretical Physics (APCTP), Korea
- Institute for Advance Study (KIAS), Korea
- College of Education, Hue University, Vietnam
- Institute of Physics, Vietnam Academy of Science & Technology, Vietnam

A Large Electron EDM and MFV

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NCTS/SJTU

Introduction

Minimal Flavor Violation

EDMs and MVF

KIAS-NCTS Joint Workshop, 2014

Introduction

Electric dipole moment: $\vec{D} = \int \rho(r)\vec{r}d^3r$ (ρ is the electric charge density).

For a fundamental particle f: $\vec{D} = d_f\vec{S}$ ($\vec{S}$ spin of the particle)

Interaction with electric field $\vec{E}$: $H_{int} = \vec{D}\cdot\vec{E}$. It violates P and T.

CTP conservation implies H_{int} violates CP.

Quantum field theory: $H_{int} = \frac{i}{2}d_f\bar{f}\sigma_{\mu\nu}\gamma_5 f F^{\mu\nu}$.

Experimental bounds

$d_e < 8.7 \times 10^{-29}$ ecm,
(ACME, arXiv:1310.7534)

$d_n < 2.9 \times 10^{-26}$ ecm
(PDG)

Standard model predictions

$d_n \sim 10^{-31}$ ecm (McKellar, Chodhury, He and Pakvasa, 1987)

$d_e \sim 10^{-44}$ ecm (Kriplovic and Pospelove, 1991)

EDM is a powerful probe of new physics beyond SM

d_e directly calculate from a given particle physics model.

But so far measurements of d_e are from molecules or atoms, such as the measurement by ACME using polar molecule thorium monoxide.

Not only de but also other CP violating interaction contribute, such as C_s ñnē γ_5 e

ACME data implies: $C_s < 5.0 \times 10^{-9}$ (if de = 0)

For neutron EDM d_n, as a composite of quarks and gluons,

more complicated as many CP violating operators may contribute.

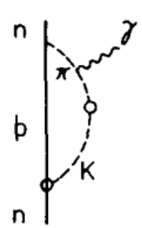
Valence quark model:

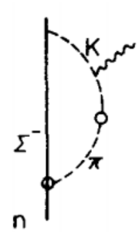
Quark EDM d_q contribution $d_n = \eta_\gamma(\frac{4}{3}d_d - \frac{1}{3}d_u)$

Quark CDM f_q contribution $d_n = \eta_c(\frac{4}{9}f_d + \frac{3}{9}f_u)e$

$\eta_{\gamma,c}$ QCD running corrections.

Hadron loop contribution:





Other CPV complicated operators. What to do?

Example the Weinberg (1989) operator:

$$O_W = -\eta_W \frac{1}{6} f_{abc} G_{a\mu\rho} G^{\rho}_{b\nu} \tilde{G}_{c\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma}$$

η_W QCD running corrections. $\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G^{\alpha\beta}$.

Naive dimensional analysis $C_o O$: $d_n \sim \frac{e}{\Lambda_\xi}(4\pi)^{2-N}\Lambda_\chi^{D-N}C_o$

D - dimension of the operator, N - number of fields,

$\Lambda_\chi = 2\pi f_\pi = 1.2$ GeV - hadron scale.

O_W contribution: $d_n = \frac{e}{4\pi}\Lambda_\chi C_W$.

Minimal Flavor Violation

There are many ways beyond SM may go.

SM: FCNC and CP violation result from mis-match between weak and mass basis.

MFV provides a model independent way of organizing new contributions beyond SM.

Basic idea: FCNC and CP violation still reside in the tree level defined Yukawa couplings.

How to realize this idea?

The renormalizable Lagrangian for flavor violation and CP violation in the SM

$$\mathcal{L}_{\mathrm{k}} \; = \; \bar{Q}_L \gamma^\mu D_\nu Q_L + \bar{U}_R \gamma^\mu D_\nu U_R + \bar{D}_R \gamma^\mu D_\nu D_R + \bar{L}_L \gamma^\mu D_\nu L_L + \bar{\nu}_R \gamma^\mu D_\nu \nu_R + \bar{E}_R \gamma^\mu D_\nu D_R \; ,$$

$$\begin{aligned}\mathcal{L}_{\mathrm{m}} \; = \; & -\bar{Q}_{i,L} \, (Y_u)_{ij} \, U_{j,R} \, \tilde{H} - \bar{Q}_{i,L} \, (Y_d)_{ij} \, D_{j,R} \, H - \bar{L}_{i,L} \, (Y_\nu)_{ij} \, \nu_{j,R} \, \tilde{H} - \bar{L}_{i,L} \, (Y_e)_{ij} \, E_{j,R} \, H \\ & - \tfrac{1}{2} \, \overline{\nu^{\mathrm{c}}_{i,R}} \, (M_\nu)_{ij} \, \nu_{j,R} \; + \; \mathrm{H.c.} \; ,\end{aligned}$$

In the basis where Y_d iand Y_e are already diagonalized,

$$Y_d \; = \; \frac{\sqrt{2}}{v} \, \hat{M}_d \; , Q_{i,L} \; = \; \begin{pmatrix} \left(V^\dagger_{\mathrm{CKM}}\right)_{ij} U_{j,L} \\ D_{i,L} \end{pmatrix} , Y_u \; = \; \frac{\sqrt{2}}{v} \, V^\dagger_{\mathrm{CKM}} \, \hat{M}_u \; ,$$

For Dirac neutrinos

$$Y_e \; = \; \frac{\sqrt{2}}{v} \, \hat{M}_e \; , L_{i,L} \; = \; \begin{pmatrix} (U_{\mathrm{PMNS}})_{ij} \, \nu_{j,L} \\ E_{i,L} \end{pmatrix} , Y_\nu \; = \; \frac{\sqrt{2}}{v} \, U_{\mathrm{PMNS}} \, \hat{m}_\nu \; ,$$

If neutrinos are Majorana fermions, neutrino mass matrix

$$\mathsf{M} \;=\; \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_\nu \end{pmatrix},$$

$M_{\rm D} = vY_\nu/\sqrt{2}$ and $M_\nu = {\rm diag}(M_1, M_2, M_3)$.
With $M_\nu \gg M_{\rm D}$, the light neutrinos' mass matrix m_ν is

$$m_\nu \;=\; -M_{\rm D} M_\nu^{-1} M_{\rm D}^{\rm T} \;=\; -\frac{v^2}{2} Y_\nu M_\nu^{-1} Y_\nu^{\rm T} \;=\; U_{\rm PMNS}\, \hat{m}_\nu\, U_{\rm PMNS}^{\rm T}\ .$$

This allows one to choose Y_ν to be

$$Y_\nu \;=\; \frac{i\sqrt{2}}{v} U_{\rm PMNS}\, \hat{m}_\nu^{1/2} O M_\nu^{1/2}\ , OO^{\rm T} = \mathbb{1}$$

Implementation of MFV

The MFV framework for quarks

L_K and L_m are formally invariant under a global group

$\mathrm{U}(3)_Q \times \mathrm{U}(3)_U \times \mathrm{U}(3)_D = G_q \times U(1)_Q \times U(1)_U \times U(1)_D$.

with $G_q = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \times \mathrm{SU}(3)_D$.

$Q_{i,L}$, $U_{i,R}$, and $D_{i,R}$ as fundamental representations of $SU(3)_{Q,U,D}$.

The Yukawa couplings $(Y_{u,d})_{ij}$ as spurions which transform as

$Q_L \to V_Q Q_L \ , \quad U_R \to V_U U_R \ , \quad D_R \to V_D D_R \ ,$

$Y_u \to V_Q Y_u V_U^\dagger \ , \quad Y_d \to V_Q Y_d V_D^\dagger, \ V_{Q,U,D} \in \mathrm{SU}(3).$

To derive nontrivial FCNC and CP-violating interactions,
one assembles an arbitrary number of the Yukawa coupling matrices to devise
$\Delta_q \sim (1+8,1,1)$, $\Delta_{u8} \sim (1,1+8,1)$, $\Delta_{d8} \sim (1,1,1+8)$,
$\Delta_u \sim (\bar{3},3,1)$, and $\Delta_d \sim (\bar{3},1,3)$
representations under G_q,
combines them with two quark fields to arrive at the G_q-invariant terms
$\bar{Q}_L \Delta_q Q_L$, $\bar{U}_R \Delta_{u8} U_R$, $\bar{D}_R \Delta_{d8} D_R$, $\bar{U}_R \Delta_u Q_L$, and $\bar{D}_R \Delta_d Q_L$,
attaches appropriate numbers of the Higgs field H and SM gauge fields to form
singlets under the SM gauge group, and also contracts all Lorentz indices.

It is simple to see that $\mathsf{A} = Y_u Y_u^\dagger$ and $\mathsf{B} = Y_d Y_d^\dagger$ transform as $(8,1,1)$.
Formally Δ_q consists of an infinite number of terms, $\Delta_q = \sum \xi_{ijk\ldots} \mathsf{A}^i \mathsf{B}^j \mathsf{A}^k \cdots$.
The MFV hypothesis requires that all the coefficients $\xi_{ijk\ldots}$ be real
complex $\xi_{ijk\ldots}$ introduce new CP-violation sources beyond that in the $Y_{u,d}$.
Looks like there are inifinite number of terms in Δ_q. But...

Using the Cayley-Hamilton identity for an arbitrary 3 x3 matrix,

$$X^3 - X^2\,\mathrm{Tr}X + X\big[(\mathrm{Tr}X)^2 - \mathrm{Tr}X^2\big]/2 - \mathbb{1}\mathrm{Det}X = 0$$

one can resum the infinite series into a finite number of terms

$$\begin{aligned}\Delta_q = {} & \xi_1\mathbb{1} + \xi_2\,\mathsf{A} + \xi_3\,\mathsf{B} + \xi_4\,\mathsf{A}^2 + \xi_5\,\mathsf{B}^2 + \xi_6\,\mathsf{AB} + \xi_7\,\mathsf{BA} + \xi_8\,\mathsf{ABA}\\ & +\xi_9\,\mathsf{BA}^2 + \xi_{10}\,\mathsf{BAB} + \xi_{11}\,\mathsf{AB}^2 + \xi_{12}\,\mathsf{ABA}^2 + \xi_{13}\,\mathsf{A}^2\mathsf{B}^2 + \xi_{14}\,\mathsf{B}^2\mathsf{A}^2\\ & +\xi_{15}\,\mathsf{B}^2\mathsf{AB} + \xi_{16}\,\mathsf{AB}^2\mathsf{A}^2 + \xi_{17}\,\mathsf{B}^2\mathsf{A}^2\mathsf{B}\ ,\end{aligned}$$

$\mathbb{1}$ is a 3×3 unit matrix.

The imaginary part of $Tr(A^2BAB^2)$ is actually equal to $(i/2)Det[A,B]$

Implies the coefficients ξ_i are in general complex and will contribute to d_e.

However, if all $\xi_{ijk...}$ are of order O(1), the imaginary part in ξ_i is of order $(i/2)Det[A,B]$.

This is suppressed by $m_\mu^2 m_\tau^2/v^4$ compared with the dominant contribution from ABA^2 term.

Other terms for imaginary part of ξ_i are from higher order terms and are further suppressed.

For Δ_{u8} and Δ_{d8}, the basic building blocks are $Y_u^\dagger Y_u$ and $Y_d^\dagger Y_d$, respectively.

These are all diagonal no new flavor- and CP-violating interactions.

Not relevant for EDM.

Our focus here being EDMs, the pertinent contributions involve Δ_u and Δ_d.

$\Delta_u = Y_u^\dagger \Delta_q$ and $\Delta_d = Y_d^\dagger \Delta_q$, with their respective coefficients ξ_r being real .

MFV for the lepton sector

the global group is $\mathrm{U}(3)_L \times \mathrm{U}(3)_\nu \times \mathrm{U}(3)_E = G_\ell \times U(1)_L \times U(1)_\nu \times U(1)_E$
with $G_\ell = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_\nu \times \mathrm{SU}(3)_E$.
$L_{i,L}$, $\nu_{i,R}$, and $E_{i,R}$ as fundamental representations of $SU(3)_{L,\nu,E}$.
Replacing V_{CKM} with $U^\dagger_{\text{PMNS}}$
employing the leptonic building blocks $\mathsf{A} = Y_\nu Y_\nu^\dagger$ and $\mathsf{B} = Y_e Y_e^\dagger$
to form the corresponding Δ_ℓ, Δ_ν, and Δ_e
transforming under G_ℓ as $(8,1,1)$, $(3,\bar{3},1)$, and $(3,1,\bar{3})$, respectively.

For Dirac neutrinos: $Y_\nu \;=\; \frac{\sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu$

For Majorana neutrinos: $Y_\nu \;=\; \frac{i\sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O M_\nu^{1/2}$,
O offers a potentially important new source of CP violation.

EDMs and MFV

At lowest order, the operators contributing directly to EDMs of fermions

$$
\begin{aligned}
O_{RL}^{(u1)} &= g'\bar{U}_R Y_u^\dagger \Delta_{qu1}\sigma_{\mu\nu}\tilde{H}^\dagger Q_L B^{\mu\nu} , & O_{RL}^{(u2)} &= g\bar{U}_R Y_u^\dagger \Delta_{qu2}\sigma_{\mu\nu}\tilde{H}^\dagger \tau_a Q_L W_a^{\mu\nu} , \\
O_{RL}^{(d1)} &= g'\bar{D}_R Y_d^\dagger \Delta_{qd1}\sigma_{\mu\nu} H^\dagger Q_L B^{\mu\nu} , & O_{RL}^{(d2)} &= g\bar{D}_R Y_d^\dagger \Delta_{qd2}\sigma_{\mu\nu} H^\dagger \tau_a Q_L W_a^{\mu\nu} , \\
O_{RL}^{(e1)} &= g'\bar{E}_R Y_e^\dagger \Delta_{\ell 1}\sigma_{\mu\nu} H^\dagger L_L B^{\mu\nu} , & O_{RL}^{(e2)} &= g\bar{E}_R Y_e^\dagger \Delta_{\ell 2}\sigma_{\mu\nu} H^\dagger \tau_a L_L W_a^{\mu\nu} ,
\end{aligned}
$$

W and B denote the usual $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

One can express the effective Lagrangian containing these operators as

$$
\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2}\left(O_{RL}^{(u1)} + O_{RL}^{(u2)} + O_{RL}^{(d1)} + O_{RL}^{(d2)} + O_{RL}^{(e1)} + O_{RL}^{(e2)}\right) + \text{H.c.} ,
$$

Λ is the MFV scale. Different operators in $\mathcal{L}_{\text{eff}}$ have different coefficients represented by ξ_r in their respective Δ's.

The contributions to EDMs proportional to

$\mathrm{Im}\big(Y_u^\dagger \Delta_{qui} V_{\rm CKM}^\dagger\big)_{kk}$, $\mathrm{Im}\big(Y_d^\dagger \Delta_{qdi}\big)_{kk}$, and $\mathrm{Im}\big(Y_e^\dagger \Delta_{\ell i}\big)_{kk}$.

$Y_{d,e}$ are diagonal, and the corresponding A's and B's Hermitian,

not all of the terms will yield nonzero contributions to fermion EDMs.

Example, $\big(Y_d^\dagger \mathsf{A}\big)_{kk} = \sqrt{2}\, m_{D_k} \mathsf{A}_{kk}/v$

for $\mathsf{A} = Y_u Y_u^\dagger$ is a real number and thus does not contribute to d_{D_k}.

Only two terms, proportional to $\mathsf{B}^2\mathsf{A}\mathsf{B}$ and $\mathsf{B}^2\mathsf{A}^2\mathsf{B}$,

are pertinent to the up-type quarks' EDMs

Only two terms, proportional to the $\mathsf{A}\mathsf{B}\mathsf{A}^2$ and $\mathsf{A}\mathsf{B}^2\mathsf{A}^2$,

are pertinent to the EDM's of down-type quarks and charged leptons.

EDMs for u and d quarks

$$
\begin{aligned}
d_u &= \frac{\sqrt{2}\, e\, v}{\Lambda^2}\, \mathrm{Im}\Big(Y_u^\dagger \Delta_{qu1} V_{\text{CKM}}^\dagger + Y_u^\dagger \Delta_{qu2} V_{\text{CKM}}^\dagger\Big)_{11} \\
&= \frac{32\, e\, m_u}{\Lambda^2}\left[\xi_{15}^{u1} + \xi_{15}^{u2} + \frac{2\big(m_c^2 + m_t^2\big)}{v^2}\big(\xi_{17}^{u1} + \xi_{17}^{u2}\big)\right] \\
&\quad \times \frac{\big(m_c^2 - m_t^2\big)\big(m_d^2 - m_s^2\big)\big(m_s^2 - m_b^2\big)\big(m_d^2 - m_b^2\big)}{v^8}\, J_q\,, \\
d_d &= \frac{\sqrt{2}\, e\, v}{\Lambda^2}\, \mathrm{Im}\Big(Y_d^\dagger \Delta_{qd1} - Y_d^\dagger \Delta_{qd2}\Big)_{11} \\
&= \frac{32\, e\, m_d}{\Lambda^2}\left[\xi_{12}^{d1} - \xi_{12}^{d2} + \frac{2\big(m_s^2 + m_b^2\big)}{v^2}\big(\xi_{16}^{d1} - \xi_{16}^{d2}\big)\right] \\
&\quad \times \frac{\big(m_s^2 - m_b^2\big)\big(m_u^2 - m_c^2\big)\big(m_c^2 - m_t^2\big)\big(m_u^2 - m_t^2\big)}{v^8}\, J_q\,,
\end{aligned}
$$

$J_q = \mathrm{Im}\big(V_{us} V_{ub}^* V_{cs}^* V_{cb}\big)$ is the Jarlskog parameter for the CKM matrix V_{CKM}.

For the electron, we get

$$
\begin{aligned}
d_e &= \frac{\sqrt{2}\, e\, v}{\Lambda^2}\, \mathrm{Im}\big(Y_e^\dagger \Delta_{\ell 1} - Y_e^\dagger \Delta_{\ell 2}\big)_{11} \\
&= \frac{\sqrt{2}\, e\, v}{\Lambda^2}\Big[\big(\xi_{12}^{\ell 1} - \xi_{12}^{\ell 2}\big)\, \mathrm{Im}\big(Y_e^\dagger \mathsf{ABA}^2\big)_{11} + \big(\xi_{16}^{\ell 1} - \xi_{16}^{\ell 2}\big)\, \mathrm{Im}\big(Y_e^\dagger \mathsf{AB}^2\mathsf{A}^2\big)_{11}\Big] \,,
\end{aligned}
$$

$\mathsf{A} = Y_\nu Y_\nu^\dagger$ and $\mathsf{B} = Y_e Y_e^\dagger$.

For Dirac neutrinos,

$$
\begin{aligned}
d_e &= \frac{32\, e\, m_e}{\Lambda^2}\left[\xi_{12}^{\ell 1} - \xi_{12}^{\ell 2} + \frac{2\big(m_\mu^2 + m_\tau^2\big)}{v^2}\big(\xi_{16}^{\ell 1} - \xi_{16}^{\ell 2}\big)\right] \\
&\quad \times \frac{\big(m_\mu^2 - m_\tau^2\big)\big(m_1^2 - m_2^2\big)\big(m_2^2 - m_3^2\big)\big(m_3^2 - m_1^2\big)}{v^8}\, J_\ell \,,
\end{aligned}
$$

$J_\ell = \mathrm{Im}\big(U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}\big)$ is the Jarlskog parameter for the PMNS matrix U_{PMNS}.

For Majorana neutrinos with O real and degenerate ν_{R_i} mass, $M_\nu = \mathrm{diag}(\hat{M}, \hat{M}, \hat{M})$

$$d_e = \frac{32\, e\, m_e}{\Lambda^2}\left[\xi_{12}^{\ell 1} - \xi_{12}^{\ell 2} + \frac{2\left(m_\mu^2 + m_\tau^2\right)}{v^2}\left(\xi_{16}^{\ell 1} - \xi_{16}^{\ell 2}\right)\right] \times \frac{\hat{M}^3\left(m_1 - m_2\right)\left(m_2 - m_3\right)\left(m_3 - m_1\right)}{v^8}\, J_\ell\,,$$

$\hat{M}^3$ dependence makes it possible to have a large electron EDM.

In the above cases, the Majorana phases in U_{PMNS} do not contribute.

If M_ν is non-degenerate, even with O real,

a non-zero Majorana phase will lead to a non-zero d_e.

With a complex O, there additional contribution to d_e from phase in O.

In this case even M_ν is degenerate, the Majorana phases can contribute to d_e.

Neutron EDM

Input values

$m_u = 0.0013 \pm 0.0004$, $m_d = 0.0027 \pm 0.0005$, $m_s = 0.055 \pm 0.017$, $m_c = 0.61 \pm 0.04$,
$m_b = 2.7 \pm 0.2$, and $m_t = 163 \pm 1$, all in GeV, at a renormalization scale $\mu \sim m_t$
$J = \left(3.02^{+0.16}_{-0.19}\right) \times 10^{-5}$

$$d_u = \frac{7.8 \times 10^{-36}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\left(\xi^u_{15} + 0.88\, \bar{\xi}^u_{17}\right), d_d = \frac{7.3 \times 10^{-30}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\left(\xi^d_{12} + 0.00024\, \xi^d_{16}\right),$$
$$d_s = \frac{-1.5 \times 10^{-28}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\left(\xi^d_{12} + 0.00024\, \xi^d_{16}\right), \xi^u_r = \xi^{u1}_r + \xi^{u2}_r, \xi^d_r = \xi^{d1}_r - \xi^{d2}_r$$

$$d_n = \eta\left(\rho^u_n d_u + \rho^d_n d_d + \rho^s_n d_s\right),$$

$\eta = 0.4$ accounts for corrections due to the QCD evolution down to the hadronic scale
$\rho^{u,d,s}_n$ depend on the model for the neutron matrix elements of $\bar{q}\sigma^{\kappa\omega}\gamma_5 q$.
$-0.78 \leq \rho^u_n \leq -0.17$, $0.7 \leq \rho^d_n \leq 2.1$, and $-0.35 \leq \rho^s_n \leq -0.01$.
(Quark model: $\rho^u_n = -1/3$, $\rho^d_n = 4/3$, $\rho^s_n = 0$.)

$$d^{\max}_n = \frac{4.9 \times 10^{-29}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\, \xi^d_{12}.$$

Not yet strong constraints.
Other contributions, such as CEDM, similar.

Electron EDM

The lepton mixing matrix U_{PMNS} can be parameterized as [5]

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} P\,,$$

$P = \mathbb{1}$ $c_{ij} = \cos\theta_{ij}$, and $s_{ij} = \sin\theta_{ij}$.

If neutrinos are Majorana particles, $P = \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$

Input values

Observable	NH	IH
$\sin^2\theta_{12}$	0.308 ± 0.017	0.308 ± 0.017
$\sin^2\theta_{23}$	$0.425^{+0.029}_{-0.027}$	$0.437^{+0.059}_{-0.029}$
$\sin^2\theta_{13}$	$0.0234^{+0.0022}_{-0.0018}$	0.0239 ± 0.0021
δ/π	$1.39^{+0.33}_{-0.27}$	$1.35^{+0.24}_{-0.39}$
$\Delta m^2_{21} = m^2_2 - m^2_1$	$(7.54^{+0.26}_{-0.22}) \times 10^{-5}\ \mathrm{eV}^2$	$(7.54^{+0.26}_{-0.22}) \times 10^{-5}\ \mathrm{eV}^2$
$\Delta m^2 = \left\|m^2_3 - (m^2_1 + m^2_2)/2\right\|$	$(2.44^{+0.08}_{-0.06}) \times 10^{-3}\ \mathrm{eV}^2$	$(2.40 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$

For Dirac neutrinos, maximize d_e^{D} for the NH or IH case is
$d_e^{\mathrm{D}} = 1.3 \times 10^{-99} \left(\xi_{12}^{\ell} + 1.0 \times 10^{-4} \xi_{16}^{\ell}\right) \left(\mathrm{GeV}^2/\Lambda^2\right) e\,\mathrm{cm}.$
This is negligible compared to the most recent experimental upper bound.

If neutrinos are Majorana fermions d_e can be sizable.
The simplest possibility that $\nu_{i,R}$ are degenerate, $M_\nu = \mathcal{M}\mathbb{1}$, and the O is real.

d_e^{M}, we obtain for $m_1 = 0$ $\left(m_3 = 0\right)$ in the NH (IH) case

$$\frac{d_e^{\mathrm{M}}}{e\,\mathrm{cm}} \;=\; 4.7\;(0.52) \times 10^{-23} \left(\frac{\mathcal{M}}{10^{15}\,\mathrm{GeV}}\right)^3 \left(\frac{\mathrm{GeV}}{\hat{\Lambda}}\right)^2,$$

$\hat{\Lambda} = \Lambda/|\xi_{12}^{\ell}|^{1/2}$ and the value of $\mathcal{M}$ is specified below.
Then $|d_e^{\mathrm{exp}}| < 8.7 \times 10^{-29}\, e\,\mathrm{cm}$ [4] implies

$$\hat{\Lambda} \;>\; 0.74\;(0.24)\;\mathrm{TeV} \left(\frac{\mathcal{M}}{10^{15}\,\mathrm{GeV}}\right)^{3/2}.$$

d_e proportional to $\mathcal{M}^3$, d_e can constrain $\hat{\Lambda}$ to a very high level with a very large $\mathcal{M}$.
However, there are restrictions on $\mathcal{M}$.
Convergence of the series in arbitrarily high powers of A and B.
If the biggest eigenvalue of A exceeds 1,
the coefficients ξ_r might not converge to finite numbers. Problematic!
However, the expansion quantities may not necessarily be A and B,
depending on the origin of MFV.
It may be that it emerges from calculations of SM loops,
naturally $\mathsf{A}/(16\pi^2)$ and $\mathsf{B}/(16\pi^2)$, the eigenvalues of A to be below $16\pi^2$.
The perturbativity condition for the Yukawa couplings, $(Y_\nu)_{ij} < \sqrt{4\pi}$,
implying a cap on the eigenvalues of A at 4π.

Benchmark values: $\mathcal{M} = 6.2 \times 10^{14}$ and 7.7×10^{15} GeV.
$\hat{\Lambda} > 360\,(120)\,\text{GeV}$ and $16\,(5)\,\text{TeV}$, respectively.
These $\mathcal{M}$ and $\hat{\Lambda}$ numbers would decrease if $m_{1(3)} > 0$.

For $M_\nu = \mathcal{M}\mathbb{1}$ and O being complex,

$\mathsf{A} = \left(2/v^2\right)\mathcal{M}\, U_{\rm PMNS}\, \hat{m}_\nu^{1/2} O O^\dagger \hat{m}_\nu^{1/2} U_{\rm PMNS}^\dagger$

In general we can write $OO^\dagger = e^{2i\mathsf{R}}$ with a real antisymmetric matrix

$$\mathsf{R} = \begin{pmatrix} 0 & r_1 & r_2 \\ -r_1 & 0 & r_3 \\ -r_2 & -r_3 & 0 \end{pmatrix} .$$

$OO^\dagger$ is not diagonal, the Majorana phases in $U_{\rm PMNS}$ also enter A if $\alpha_{1,2} \neq 0$.

We focus first on the CP-violating effect of O by setting $\alpha_{1,2} = 0$.

Pick $r_{1,2,3} = \rho$, employ the experimental central values, we have

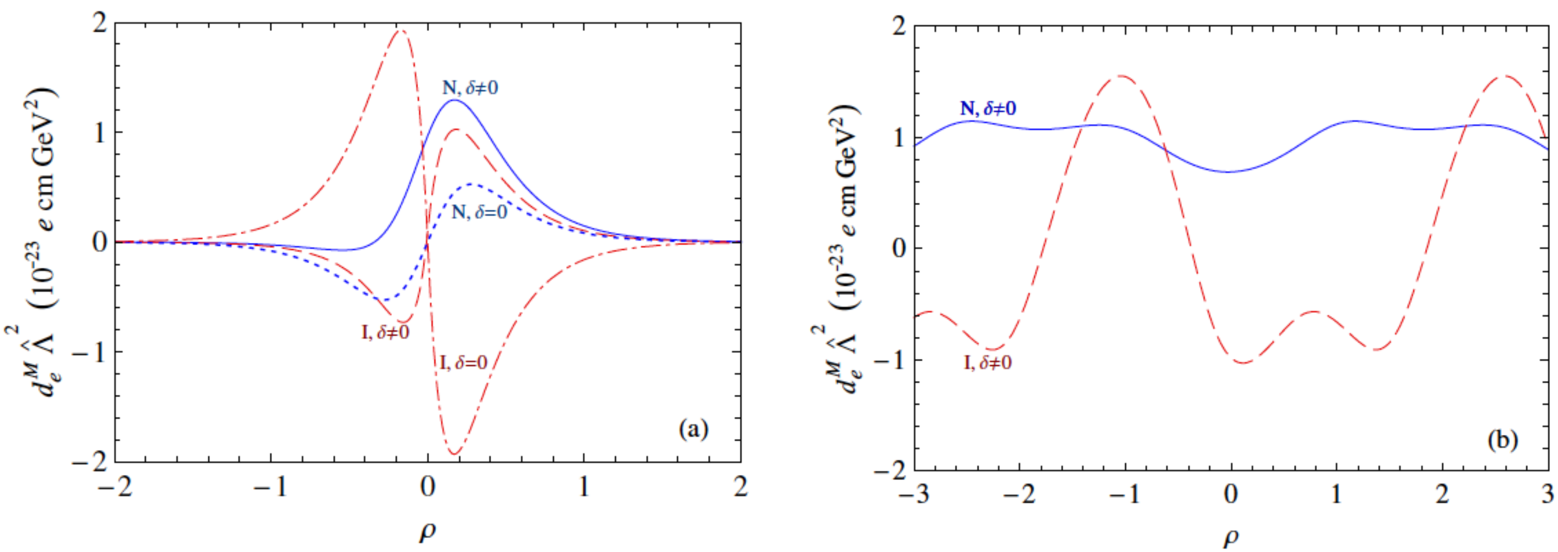


FIG. 1: Dependence of $d_e^{\rm M}$ times $\hat{\Lambda}^2 = \Lambda^2/\xi_{12}^\ell$ on ρ in the absence of Majorana phases, $\alpha_{1,2} = 0$, for (a) degenerate $\nu_{i,R}$ and complex O and (b) nondegenerate $\nu_{i,R}$ and real O, as explained in the text. In all figures, the label N (I) refers to the NH (IH) case with $m_{1(3)} = 0$.

Effect of Majorana Phase

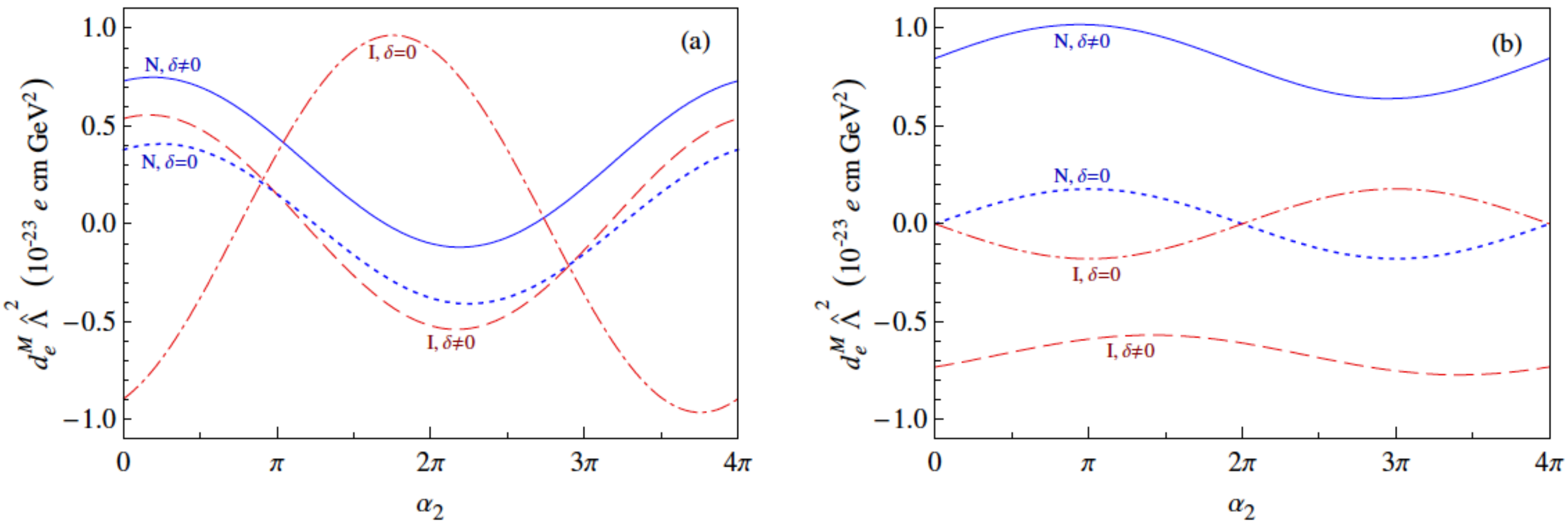


FIG. 2: Dependence of $d_e^{\mathrm{M}}\hat{\Lambda}^2$ on α_2 for $\alpha_1 = 0$ and $\rho = 0.5$ with (a) degenerate $\nu_{i,R}$ and complex O and (b) nondegenerate $\nu_{i,R}$ and real O, as explained in the text.

Constraint on CP-violating electron-neutron interaction $\mathcal{L}_{en} = -i\left(C_S\, G_{\rm F}/\sqrt{2}\right) \bar{e}\gamma_5 e\, \bar{n}n$.

The lowest-order MFV operators contributing to C_S are given by

$$\mathcal{L}_{\ell q} \;=\; \frac{1}{\Lambda^2}\, \bar{E}_R Y_e^\dagger \Delta'_{\ell 1} L_L\, \bar{U}_R Y_u^\dagger \Delta_{qu}\, i\tau_2 Q_L \,+\, \frac{1}{\Lambda^2}\, \bar{E}_R Y_e^\dagger \Delta'_{\ell 2} L_L\, \bar{Q}_L \Delta_{qd}^\dagger Y_d D_R \,+\, {\rm H.c.}$$

To determine C_S, we need to know the matrix elements $\langle n|m_q \bar{q} q|n\rangle = g_q^n\, \bar{u}_n u_n v$.

For $M_\nu = \mathcal{M}\mathbb{1}$ and O being real

$$C_S \;=\; \frac{16\sqrt{2}\, J_\ell\, m_e \mathcal{M}^3}{\Lambda^2 G_{\rm F}\, v^9} \left(m_\mu^2 - m_\tau^2\right)\left(m_1 - m_2\right)\left(m_2 - m_3\right)\left(m_3 - m_1\right)$$
$$\times \left[\left(g_d^n + g_s^n + g_t^n\right) \xi_{12}^{\ell 2} - \left(g_u^n + 2g_t^n\right) \xi_{12}^{\ell 1}\right] .$$

The experimental bound $|C_S| < 5.9 \times 10^{-9}$ reported by ACME then implies,

if $\mathcal{M} \simeq 6 \times 10^{14}\,{\rm GeV}$ and the maximal values of g_q^n from Ref. [19]

$$\hat{\Lambda} \;>\; 0.24\ (0.077)\ {\rm GeV}$$

in the NH (IH) case with $m_{1(3)} = 0$.

This is far less stringent than the restriction from d_e directly.

- In MFV d_e crucially depends on whether neutrinos are Dirac or Majorana particles. d_e can reach the experimental bound for Majorana neutrinos and the scale of minimal flavor violation is a few hundred GeV or higher.

- There are new CP violating effects on d_e in the Yukawa couplings of the right-handed neutrinos. These new sources can have dramatic effects for d_e.